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A NOMOGRAM METHOD FOR ESTIMATING THE ENERGY PRODUCED BY WIND TURBINE GENERATORS

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Abstract—A simple nomogram is constructed to estimate the power generated by a wind turbine generator (WTG) operated at near maximum efficiency using optimum tip-speed ratio between cut-in and rated wind speed, and at constant power using optimum pitch control between rated and cut-out wind speed. The nomogram is based on information that is readily available for commercial WTGs as well as some simple statistical quantities for the wind at the site. When the wind speed is described by a Weibull distribution, the power of a WTG is estimated in terms of three generalized non-dimensional parameters. When a Rayleigh distribution is employed only two parameters are necessary. A second nomogram is also developed for those less common cases where a small correction of the results of the first nomogram is needed. A mathematical analysis is presented which allows for the construction of single chart nomograms without sacrificing the necessary accuracy. Two application examples demonstrate the degree of accuracy achieved by the nomograms and the advantages they offer for parametric analyses as regards convenience and labor. © 2002 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

Development of techniques for accurately assessing the wind power potential of a site is gaining increased importance. This is because of the fact that planning and establishment of a wind energy system depend upon factors like variation of wind speed distribution e.g. mean wind speed, $\bar{v}$, and its standard deviation, σ, and characteristic operational speeds of turbine viz. cut-in velocity, v_{ci}, rated velocity, v_r, and cut-out velocity, v_{co}.

A number of studies have dealt with the fitting of wind speed distributions (Nfaouy *et al*., 1998; Garcia *et al*., 1998) and site matching of wind turbine generators (Pallazzer, 1995; Jangamshetti and Rau, 1999; Feretic *et al*., 1999). In those studies, the Weibull or the Rayleigh distribution was employed for the description of the wind speed variation. The WTG average power was estimated from the integration with respect to time of the product of the wind speed probability density function times the turbine's power characteristic curve. The complexity of this integration does not allow for an exact analytical solution but requires extensive numerical calculations. If, in addition, one has to perform a systematic parametric investigation, the effective computer implementation might be particularly cumbersome since then a quite complex numerical code is required.

Nomograms, sometimes referred to as alignment charts, are graphical calculators that are closely related to the old-fashioned slide rule. Despite the rapid advance of computational power, they are still of practical use in many situations where it is important to make fast and simple estimates (Garg *et al*., 1998).

In this paper, suitable nomograms are constructed in order to estimate the WTG average power from basic quantities of the wind speed distribution and some WTG technical data. These nomograms correlate the following generalized non-dimensional parameters $v_{ci}/\bar{v}$, $v_r/\bar{v}$, $v_{co}/\bar{v}$, $\sigma/\bar{v}$ with the so-called capacity factor τ, which, in essence, describes the integration mentioned above. The nomograms are constructed according to a formal mathematical methodology (Menzel, 1960), which enables the effective representation of the extensive interrelationships among the aforementioned parameters in a single comprehensive graph. Then, the estimation of τ is realized, fast and easy, just by drawing the appropriate straight-line through the nomograms' curves.

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2. MATHEMATICAL FORMULATION

2.1. Turbine power output

For wind turbine machines that operate at near maximum efficiency using optimum tip-speed ratio between cut-in and rated wind speed and at constant power using optimum pitch control between rated and cut-out wind speed, the power output P is well described by the formula (Twidell and Weir, 1986):

$$P = \begin{cases} 0 & : v < v_{ci} \\ \mu v^3 - \nu P_r & : v_{ci} \le v < v_r \\ P_r & : v_r \le v < v_{co} \\ 0 & : v_{co} \le v \end{cases} \tag{1}$$

where μ and ν are constants, v the wind speed and P_r the rated wind turbine power. Turbines following the above operation scheme can effectively extract wind power while at the same time maintain safe operation and for this reason such turbines are quite common among large scale manufacturers (Ackerman and Soder, 2000). It must be mentioned though that other types of turbines are also in commercial use, e.g. stall controlled, which do not obey Eq. (1). From the conditions at cut-in speed where $P = 0$ and at rated power where $P = P_r$, the constants μ and ν can be determined in terms of v_{ci}, v_r and P_r as $\mu = P_r/(u_r^3 - u_{ci}^3)$ and $\nu = u_{ci}^3/(u_r^3 - u_{ci}^3)$. Thus, Eq. (1) in the region $v_{ci} \le v < v_r$ becomes

$$P = \frac{v^3 - v_{ci}^3}{v_r^3 - v_{ci}^3} \cdot P_r \tag{2}$$

2.2. Wind statistical models

The wind speed frequency curve can be modeled by a continuous mathematical function, namely, the probability density function $f(v)$. In wind power studies the Weibull and Rayleigh probability density functions are very popular and for this, it was decided to also use them here.

The Weibull probability density function is a special case of a Pearson type III or generalized Gamma distribution with two parameters. Wind speeds v distributed according to the Weibull probability density function are represented as follows

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-(v/c)^k} \tag{3}$$

where k is the *shape* parameter and c is the *scale* parameter of the distribution.

The Rayleigh distribution is a subset of the Weibull distribution (with $k = 2$) and has only one adjustable parameter. This makes it simpler to use in calculations. The Rayleigh formula for a wind speed distribution is

$$f(v) = \frac{2v}{c^2} e^{-(v/c)^2} \tag{4}$$

There exist several methods for determining the parameters k and c (Justus, 1978). If the mean speed, $\bar{v}$, and the standard deviation, σ, of the wind speed for a site are known, then the following equations can be used to find c and k

$$\overline{v^n} = \int_0^\infty v^n \cdot \frac{k}{c} \cdot \left(\frac{v}{c}\right)^{k-1} \cdot e^{-(v/c)^k} \cdot dv = c^n \cdot \Gamma(1 + n/k) \tag{5}$$

where Γ is the gamma function and n can be an integer or fractional number (Twidell and Weir, 1986). The variance of the wind speed is defined as

$$\sigma^2 = \overline{v^2} - \bar{v}^2 \tag{6}$$

If we put $n = 1$ and 2 in Eqs. (5) and (6), after some algebra we end up with the following equations

$$\left(\frac{\sigma}{\bar{v}}\right)^2 = \frac{\Gamma(1 + 2/k)}{(\Gamma(1 + 1/k))^2} - 1 \tag{7}$$

$$c = \frac{\bar{v}}{\Gamma(1 + 1/\kappa)} \tag{8}$$

An acceptable approximation for the values of k is given by the relation

$$k = \left(\frac{\sigma}{\bar{v}}\right)^{-1.090} \tag{9}$$

Justus (1978) communicated the relation $k = (\sigma/\bar{v})^{-1.086}$, which is in close agreement with Eq. (9). Fitting the Weibull distribution to measured data often gives values of k between about 1.6 and 3.0 (Boweden *et al.*, 1983), which corresponds to the ratio $\sigma/\bar{v}$ being varied between 0.35 and 0.65. In this case, c can be approximated to within 1% by

$$c \approx \frac{2 \cdot \bar{v}}{\sqrt{\pi}} \tag{10}$$

For a wider k interval or for accuracy better than 0.01%, c can be represented by

$$c \approx \bar{v} \frac{k^{2.6674}}{0.184 + 0.816 \cdot k^{2.73855}} \tag{11}$$

Apparently, the numerical solution of Eqs. (7) and (8) holds for any value of $\sigma/\bar{v}$.

2.3. *Estimation of the capacity factor*

The *capacity factor*, τ is defined as the ratio of the average power output over a time period versus the rated power

$$\tau = \frac{P_{\text{aver}}}{P_{\text{r}}} \tag{12}$$

The average power output of the wind turbine is the power produced at every particular wind speed multiplied by the probability of this wind speed and integrated over all possible wind speeds. In integral form this is expressed as

$$P_{\text{aver}} = \int_0^{\infty} P(\upsilon) \cdot f(\upsilon)\, \mathrm{d}\upsilon \tag{13}$$

Employing Eqs. (1) and (2) $P(\upsilon)$, the average power output can be written as

$$P_{\text{aver}} = \int_{\upsilon_{\text{ci}}}^{\upsilon_{\text{r}}} \frac{\upsilon^3 - \upsilon_{\text{ci}}^3}{\upsilon_{\text{r}}^3 - \upsilon_{\text{ci}}^3} \cdot P_{\text{r}} \cdot \frac{k}{c} \left(\frac{\upsilon}{c}\right)^{k-1} \mathrm{e}^{-(\upsilon/c)^k}\, \mathrm{d}\upsilon + \int_{\upsilon_{\text{r}}}^{\upsilon_{\text{co}}} P_{\text{r}} \cdot \frac{k}{c} \left(\frac{\upsilon}{c}\right)^{k-1} \mathrm{e}^{-(\upsilon/c)^k}\, \mathrm{d}\upsilon \tag{14}$$

Introduction of the dummy variable $x = \upsilon/\sigma\bar{\upsilon}$ and further integration with respect to x results in the following relation

$$P_{\text{aver}} = \left(\frac{3}{k} \cdot \frac{\upsilon_{\text{ci}}^3 \cdot E_{(k-3)/k}\left((\upsilon_{\text{ci}}/c)^k\right) - \upsilon_{\text{r}}^3 \cdot E_{(k-3)/k}\left((\upsilon_{\text{r}}/c)^k\right)}{\upsilon_{\text{r}}^3 - \upsilon_{\text{ci}}^3} - \mathrm{e}^{-(\upsilon_{\text{co}}/c)^k} \right) \cdot P_{\text{r}} \tag{15}$$

where

$$E_n(z) = \int_1^{\infty} \frac{\mathrm{e}^{-zt}}{t^n}\, \mathrm{d}t$$

In the case that a Rayleigh distribution is used for the wind speed then $k = 2$ and $c = 2\bar{\upsilon}/\sqrt{\pi}$ and Eq. (15) becomes

$$P_{\text{aver}} = \left(\frac{3}{2} \cdot \frac{\upsilon_{\text{ci}}^3 \cdot E_{-1/2}\left(\pi\upsilon_{\text{ci}}^2/4\bar{\upsilon}^2\right) - \upsilon_{\text{r}}^3 \cdot E_{-1/2}\left(\pi\upsilon_{\text{r}}^2/4\bar{\upsilon}^2\right)}{\upsilon_{\text{r}}^3 - \upsilon_{\text{ci}}^3} - \mathrm{e}^{-(\pi\upsilon_{\text{co}}^2/4\bar{\upsilon}^2)} \right) \cdot P_{\text{r}} \tag{16}$$

In both Eqs. (15) and (16), the capacity factor is represented by the expressions in the long parentheses.

3. NOMOGRAM DEVELOPMENT

A nomogram furnishes a graphical procedure solving certain types of equations, primarily those containing three variables, say α, β and γ. If any two quantities are known the third follows directly from the conditioning equation between the three parameters. The determinant

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \tag{17}$$

represents the equation:

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_3}{x_2 - x_3} \tag{18}$$

The three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear because the two segments possess identical slopes and because they have a point (x_2, y_2) in common. Now, suppose that we can write

$$\begin{aligned} x_1 &= \phi_1(\alpha) & y_1 &= \psi_1(\alpha) \\ x_2 &= \phi_2(\beta) & y_2 &= \psi_2(\beta) \\ x_3 &= \phi_3(\gamma) & y_3 &= \psi_3(\gamma) \end{aligned} \tag{19}$$

Eqs. (19) define three curves, in parametric form, whose points we can label in terms of our basic parameters α, β and γ. Thus if we know α and β, for example, a straight line from the appropriate points on the α and β curves will intersect the γ curve at the value of γ that satisfies the equations. Consequently, to represent an equation nomographically we must find first an equivalent determinant, which we must transform next by standard rules until we obtain an equation in the form of (17), viz.

$$\begin{vmatrix} \phi_1(\alpha) & \psi_1(\alpha) & 1 \\ \phi_2(\beta) & \psi_2(\beta) & 1 \\ \phi_3(\gamma) & \psi_3(\gamma) & 1 \end{vmatrix} = 0 \tag{20}$$

Although Eq. (20) satisfies the initial condition, (17), the diagram resulting from it is by no means the most general possible. Let a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 and a_9 represent nine arbitrary constants. Using the transformation properties of a zero-valued determinant we can obtain the general form, whose expansion is equivalent to Eq. (20)

$$\begin{vmatrix} \dfrac{a_1 \cdot \phi_1 + a_2 \cdot \psi_1 + a_3}{a_7 \cdot \phi_1 + a_8 \cdot \psi_1 + a_9} & \dfrac{a_4 \cdot \phi_1 + a_5 \cdot \psi_1 + a_6}{a_7 \cdot \phi_1 + a_8 \cdot \psi_1 + a_9} & 1 \\ \dfrac{a_1 \cdot \phi_2 + a_2 \cdot \psi_2 + a_3}{a_7 \cdot \phi_2 + a_8 \cdot \psi_2 + a_9} & \dfrac{a_4 \cdot \phi_2 + a_5 \cdot \psi_2 + a_6}{a_7 \cdot \phi_2 + a_8 \cdot \psi_2 + a_9} & 1 \\ \dfrac{a_1 \cdot \phi_3 + a_2 \cdot \psi_3 + a_3}{a_7 \cdot \phi_3 + a_8 \cdot \psi_3 + a_9} & \dfrac{a_4 \cdot \phi_3 + a_5 \cdot \psi_3 + a_6}{a_7 \cdot \phi_3 + a_8 \cdot \psi_3 + a_9} & 1 \end{vmatrix} = 0 \tag{21}$$

By properly choosing the constants a_1 to a_9 one can adjust the positions and scales of the α, β, γ curves in order to achieve maximum accuracy for the desired range of variables. The analysis above is described by Menzel (1960).

The application of the aforementioned procedure for the present case is as follows. The capacity factor from Eq. (15) can be written as

$$\tau = \frac{3}{k} \cdot \frac{v_{ci}^3 \cdot E_{(k-3)/k}\left((v_{ci}/c)^k\right) - v_r^3 \cdot E_{(k-3)/k}\left((v_r/c)^k\right)}{v_r^3 - v_{ci}^3} - e^{-(v_{co}/c)^k} = \tau_1 - \tau_2 \quad (22a)$$

where

$$\tau_1 = \frac{3}{k} \cdot \frac{v_{ci}^3 \cdot E_{(k-3)/k}\left((v_{ci}/c)^k\right) - v_r^3 \cdot E_{(k-3)/k}\left((v_r/c)^k\right)}{v_r^3 - v_{ci}^3} \quad (22b)$$

and

$$\tau_2 = e^{-(v_{co}/c)^k} \quad (22c)$$

The term τ_1 is the one that heavily scores on τ. In ordinary cases (where mean wind speed is considerably lower than cut-out speed, see below) the term τ_2 is negligible and can be safely ignored. Division of both sides of (22b) by $\bar{v}^3$ and further rearrangement gives

$$\frac{\tau_1}{3} \cdot \left[\left(\frac{v_r}{\bar{v}}\right)^3 - \left(\frac{v_{ci}}{\bar{v}}\right)^3\right] + \left(\frac{v_r}{\bar{v}}\right)^3 \cdot \frac{E_{(k-3)/k}\left((v_r/c)^k\right)}{k} - \left(\frac{v_{ci}}{\bar{v}}\right)^3 \cdot \frac{E_{(k-3)/k}\left((v_{ci}/c)^k\right)}{k} = 0 \quad (23)$$

which in a simplified form can be written as

$$\frac{\alpha}{3} \cdot (\gamma^3 - \beta^3) + \gamma^3 \cdot f_e(\gamma, \sigma') - \beta^3 \cdot f_e(\beta, \sigma') = 0 \quad (24)$$

where

$$\alpha = \tau_1,\ \beta = \left(\frac{v_{ci}}{\bar{v}}\right),\ \gamma = \left(\frac{v_r}{\bar{v}}\right),\ \sigma' = \left(\frac{\sigma}{\bar{v}}\right)$$

and

$$f_e(\beta, \sigma') = \frac{E_{(k-3)/k}\left((v_{ci}/c)^k\right)}{k},$$

$$f_e(\gamma, \sigma') = \frac{E_{(k-3)/k}\left((v_r/c)^k\right)}{k}$$

The arguments of the last two functions, (β, σ') and (γ, σ'), portray the dependence shown in Eqs. (9), (10) and (11). Thus, τ_1 is eventually a function of $(v_{ci}/\bar{v})$, $(v_r/\bar{v})$, and $(\sigma/\bar{v})$.

To generate a nomogram, Eq. (24) must be written in a de-coupled parametric matrix form as Eq. (20). A suitable matrix is derived as follows

$$\begin{pmatrix} \frac{\alpha}{3} & 0 & 1 \\ -\beta^3 \cdot f_e(\beta, \sigma') & 1 & \beta^3 \\ -\gamma^3 \cdot f_e(\gamma, \sigma') & 1 & \gamma^3 \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{3} & 0 & 1 \\ -f_e(\beta, \sigma') & \frac{1}{\beta^3} & 1 \\ -f_e(\gamma, \sigma') & \frac{1}{\gamma^3} & 1 \end{pmatrix} = 0 \quad (25)$$

Consequently, the parametric curves of the nomogram are given by the equations

$$\begin{aligned} \phi_1(\alpha) &= \frac{\alpha}{3} & \psi_1(\alpha) &= 0 \\ \phi_2(\beta) &= -f_e(\beta, \sigma') & \psi_2(\beta) &= \frac{1}{\beta^3} \\ \phi_3(\gamma) &= -f_e(\gamma, \sigma') & \psi_3(\gamma) &= \frac{1}{\gamma^3} \end{aligned} \quad (26)$$

To increase the clarity and, therefore, the accuracy of the nomogram, a parametric analysis is performed to derive the best values for the coefficients a_i of Eq. (21). A satisfactory accuracy is achieved with the following coefficient values: $a_1 = a_5 = 1$; $a_3 = a_4 = a_6 = a_7 = 0$; $a_2 = 0.8$; $a_8 = 0.7$; $a_9 = 0.4$. Fig. 1 displays the nomogram representing the final outcome for the estimation of τ_1.

For those cases that the term τ_2 cannot be ignored, another nomogram is constructed to estimate it graphically. The term τ_2, otherwise referred to as correction factor, is given in Eq. (22c) as a function of, v_{co}, c and k. Following arguments similar to those advanced as regards τ_1, it is easily concluded that τ_2 is eventually a function of $v_{co}/\bar{v}$ and $(\sigma/\bar{v})$. Taking two times the logarithm of Eq. (22c) gives

$$\ln(-\ln(\tau_2)) - k \cdot \ln\left(\frac{v_{co}}{\bar{v}}\right) + k \cdot \ln\left(\frac{c}{\bar{v}}\right) = 0 \quad (27)$$

or, in a more generic form

$$\ln(-\ln(\tau_2)) - f_k(\sigma') \cdot \ln\left(\frac{v_{co}}{\bar{v}}\right) + f(\sigma') = 0 \quad (28)$$

where

$$f(\sigma') = k \ln\left(\frac{c}{\bar{v}}\right) \text{ and } f_k(\sigma') = (\sigma')^{-1.090}$$

Writing Eq. (28) in the form of a nomogram matrix results in

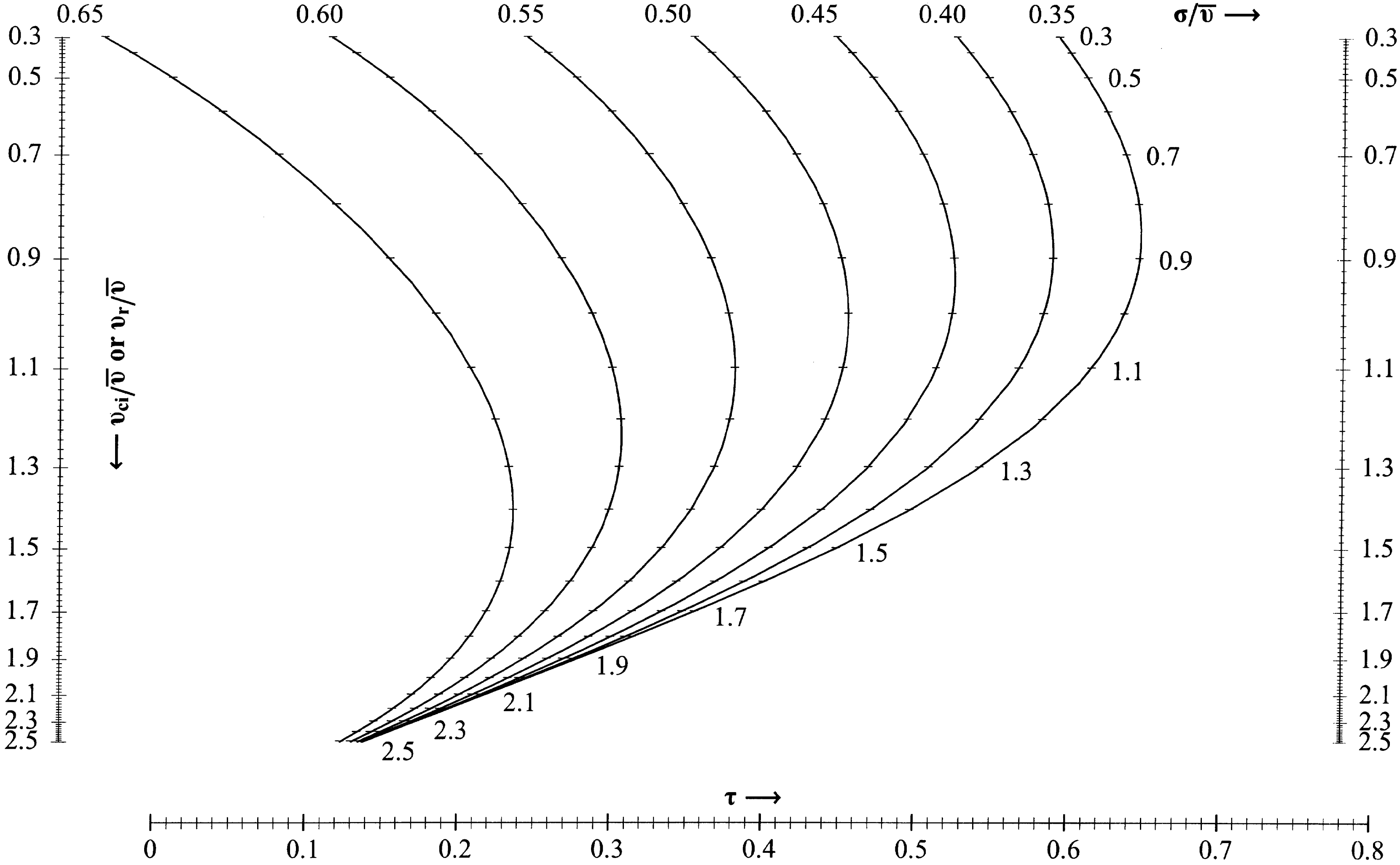

Fig. 1. Nomogram for the estimation of the term τ_1 of the capacity factor for a wind turbine generator.

$$\begin{pmatrix} \ln(-\ln(\tau_2)) & 0 & 1 \\ -\ln\left(\frac{v_{co}}{\bar{v}}\right) & 1 & 0 \\ -f(\sigma') & f_k(\sigma') & 1 \end{pmatrix} = \begin{pmatrix} \ln(-\ln(\tau_2)) & 0 & 1 \\ -\ln\left(\frac{v_{co}}{\bar{v}}\right) & 1 & 1 \\ -f(\sigma') & f_k(\sigma') & 1+f_k(\sigma') \end{pmatrix} = \begin{pmatrix} \ln(-\ln(\tau_2)) & 0 & 1 \\ -\ln\left(\frac{v_{co}}{\bar{v}}\right) & 1 & 1 \\ -\frac{f(\sigma')}{1+f_k(\sigma')} & \frac{f_k(\sigma')}{1+f_k(\sigma')} & 1 \end{pmatrix} = 0 \quad (29)$$

Accordingly, the parametric curves of the nomogram are described by the equations

$$\phi_1(\tau_2) = \ln(-\ln(\tau_2)) \quad \psi_1(\tau_2) = 0$$
$$\phi_2\left(\frac{v_{co}}{\bar{v}}\right) = -\ln\left(\frac{v_{co}}{\bar{v}}\right) \quad \psi_2\left(\frac{v_{co}}{\bar{v}}\right) = 1$$
$$\phi_3(\sigma') = -\frac{f(\sigma')}{1+f_k(\sigma')} \quad \psi_3(\sigma') = \frac{f_k(\sigma')}{1+f_k(\sigma')} \quad (30)$$

The respective nomogram for τ_2 is displayed in Fig. 2. It must be noted that the clarity of Fig. 2 is considered adequate just as generated from the parametric curves (30). So, no extra effort is undertaken to derive values for the coefficients a_i of Eq. (21). In Fig. 2 it is readily seen that τ_2 can be safely ignored for cases where ($v_{co}/\bar{v} \geq \sim 1.8$ for $\sigma/\bar{v} \approx 0.35$, $v_{co}/\bar{v} \geq \sim 2.1$ for $\sigma/\bar{v} \approx 0.45$, $v_{co}/\bar{v} \geq \sim 2.45$ for $\sigma/\bar{v} \approx 0.55$ and $v_{co}/\bar{v} \geq \sim 2.85$ for $\sigma/\bar{v} \approx 0.65$) with an approximate error in power prediction of less than 1%. These cases are the most frequently encountered in applications.

To this end, the average power generated by the WTG is estimated as

$$P_{aver} = \tau \cdot P_r = (\tau_1 - \tau_2) \cdot P_r \quad (31)$$

whereas the energy produced over a period of application Δt is given as

$$E = P_{aver} \cdot \Delta t \quad (32)$$

4. APPLICATION EXAMPLES

(a) The first example aims to demonstrate the convenience in using the nomograms but also how fast one can conduct a parametric analysis in order to investigate the influence of the various quantities to the produced power. Fig. 3 (replica of Fig. 1) depicts the procedure for the estimation of the average power, P_{aver}, for a wind turbine with the following characteristics: P_r, $v_{ci} = 3.5$ m/s, $v_r = 12$ m/s and $v_{co} > 25$ m/s. The estimation is performed for four different wind fields

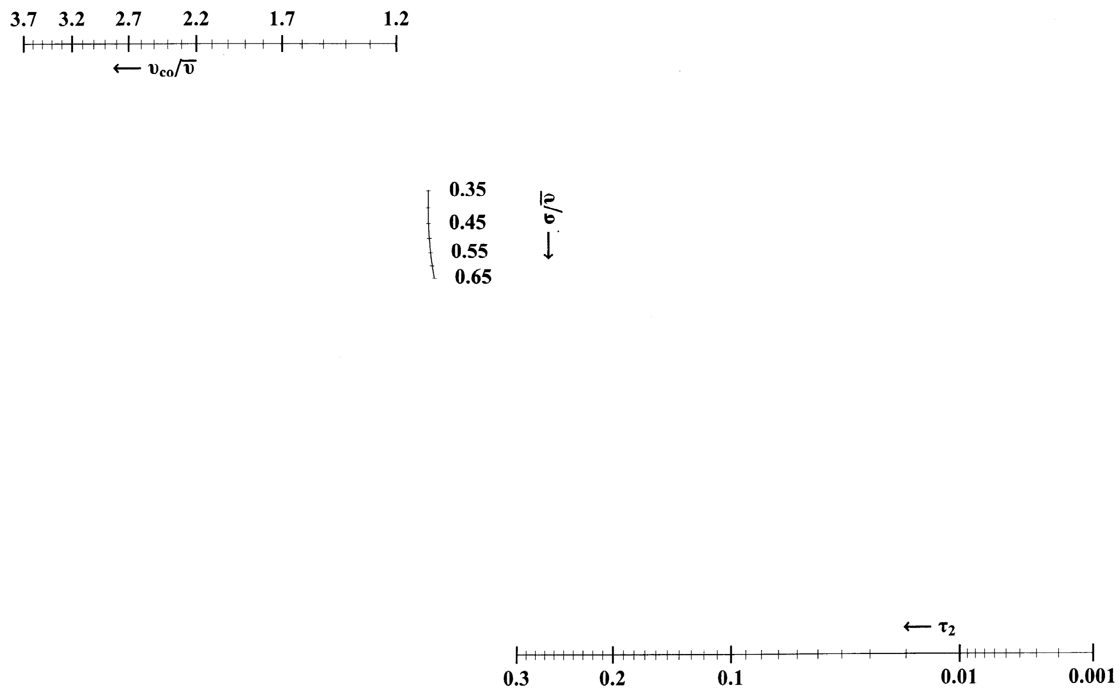

Fig. 2. Nomogram for the estimation of the term τ_2 of the capacity factor for a wind turbine generator.

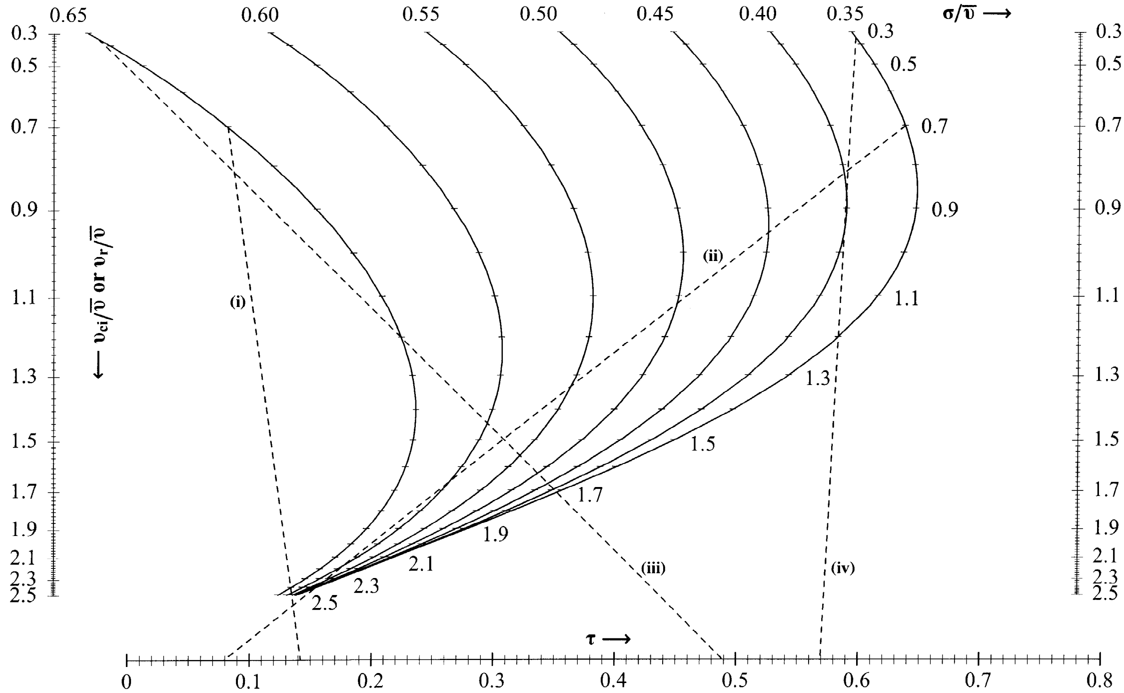

Fig. 3. Investigation of the performance of a wind turbine generator under different wind speed conditions by using the nomogram for the estimation of τ_1.

having the following characteristics: (i) $\bar{v} = 5$ m/s, $\sigma' = 0.65$, (ii) $\bar{v} = 5$ m/s, $\sigma' = 0.35$, (iii) $\bar{v} = 10$ m/s, $\sigma' = 0.65$ and (iv) $\bar{v} = 10$ m/s, $\sigma' = 0.35$. The steps for the estimation of P_{aver} for case (i) are as follows. Firstly, the curve corresponding to $\sigma' = 0.65$ is selected. Then, the points $v_{ci}/\bar{v} = 3.5/5 = 0.7$ and $v_r/\bar{v} = 12/5 = 2.4$ are marked on this curve and a straight line is drawn connecting these two points. This line is extrapolated until the τ_1-axis. The value of the intercept multiplied by P_r, the rated wind turbine power, gives P_{aver}.

From Fig. 3 and Eq. (31) the values obtained for P_{aver}, are: (i) $0.142P_r$, (ii) $0.08P_r$, (iii) $0.49P_r$ and (iv) $0.57P_r$. Apparently, when the wind mean speed is quite low, close to v_{ci}, a moderate increase of the standard deviation leads to significantly higher powers (cases i and ii). This trend is reversed when the wind mean speed is relatively large, close to v_r, (cases iii and iv). In the above calculations it is assumed for simplicity that the correction factor τ_2 is negligible.

(b) To check on the degree of accuracy of the developed nomograms, a comparison is made between the capacity factor values obtained graphically and those calculated by numerically solving the descriptive equations. Furthermore, a comparison is made with the numerical WTG site-matching procedure employed by Jangamshetti and Rau (1999), which analysed real technical and field data for several modern turbine generators. However, the communicated data are in a format not amenable to direct comparison with the present analysis so it was decided instead to compare with the numerical results of Jangamshetti and Rau since their estimated annual capacity factors are in fair agreement with experimental values. For this, the same wind turbine generators and wind speed data that have been used by Jangamshetti and Rau (1999), are also employed here. The annual mean speed and standard deviation of the wind are: $\bar{v} = 8.7$ m/s and $\sigma = 3.96$ m/s. Table 1 presents the basic technical data of the different commercial wind turbines.

Using the nomograms of Figs. 1 and 2, the annual capacity factor of the different wind turbines is estimated graphically. These values are displayed in Table 2 next to the values calculated numerically from the original integrated equations and also the values communicated by Jangamshetti and Rau (1999).

It is clearly seen that the estimations from the nomograms are in very close proximity with the numerical values manifesting the accuracy and reliability of the nomogram method. Evidently, the accuracy provided by the nomograms may be

Table 1. Technical data of different commercial wind turbines

No.	Turbine name and model no.	Rated power (kW)	Characteristic speed (m/s)		
			Cut-in, v_{ci}	Rated, v_r	Cut-out, v_{co}
1	Vestas, V-27	225	3.5	13.5	25.0
2	Vestas, V-25	200	3.5	13.8	25.0
3	WD-34	400	5.0	13.0	25.0
4	MWT-275	275	5.0	12.9	24.0
5	MWT-300	300	5.5	12.0	24.0
6	Alder-25	165	3.5	13.0	20.0
7	Nordex-250	250	3.0	13.0	25.0
8	Nordex-250	250	4.0	13.0	25.0
9	Nordex-150	150	3.0	10.0	25.0
10	Nordex-150	150	4.0	10.0	25.0
11	BE-100/25	150	4.0	14.0	28.0
12	Mod-0	100	4.3	7.7	17.9

Table 2. Annual capacity factors of different commercial wind turbines

No.	$v_{ci}/\bar{v}$	$v_r/\bar{v}$	$v_{co}/\bar{v}$	τ_1	τ_2	τ	τ numerical	τ from Jangamshetti and Rau (1999)
1	0.403	1.554	2.877	0.350	0.000	0.350	0.350	0.360
2	0.403	1.588	2.877	0.335	0.000	0.335	0.335	0.345
3	0.575	1.496	2.877	0.360	0.000	0.360	0.358	0.384
4	0.575	1.485	2.762	0.365	0.000	0.365	0.363	0.389
5	0.633	1.381	2.762	0.405	0.000	0.405	0.403	0.439
6	0.403	1.496	2.302	0.380	0.005	0.375	0.373	0.383
7	0.345	1.496	2.877	0.380	0.000	0.380	0.381	0.388
8	0.460	1.496	2.877	0.370	0.000	0.370	0.372	0.387
9	0.345	1.151	2.877	0.570	0.000	0.570	0.569	0.579
10	0.460	1.151	2.877	0.555	0.000	0.555	0.555	0.576
11	0.460	1.611	3.222	0.320	0.000	0.320	0.320	0.334
12	0.495	0.886	2.060	0.700	0.016	0.684	0.682	0.710

over and above deviations caused by uncertainty in input parameters, i.e. turbine technical characteristics and wind field data. Moreover, the present results for τ are in good agreement with the results of Jangamshetti and Rau (1999) despite the fact that they employed a slightly different equation to describe the wind turbine performance, other than Eq. (1). This lends further support to the applicability of the developed nomograms.

5. CONCLUSIONS

A single-chart nomogram is developed for estimating the power produced by a wind turbine generator (WTG) operated at near maximum efficiency using optimum tip-speed ratio between cut-in and rated wind speed and at constant power using optimum pitch control between rated and cut-out wind speed. For some rare (wind field) cases, an auxiliary nomogram is also constructed to provide minor modifications to the results of the first nomogram. Each nomogram correlates in a comprehensive way different generalized non-dimensional parameters, which are formed from some basic WTG operational data, the cut-in speed (v_{ci}), the rated speed (v_r) and the cut-out speed (v_{co}), as well as the mean wind speed at the site ($\bar{v}$) and its standard deviation (σ). A formal mathematical procedure is proposed for the construction of single-chart nomograms with exceptional scaling accuracy. Two application examples delineate that the developed nomograms describe successfully the extensive interrelationships among the governing parameters.

A major advantage of the proposed nomograms is that they allow for a quick, yet reliable, estimate of a design case. An additional advantage of the nomograms is that an overview and sensitivity impression of the results can be easily derived whereas conventional computer calculations can only give the result in one single point and one has to make a lot of runs before one can see tendencies.

NOMENCLATURE

a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9	arbitrary constants
f_e, f	functions
E	energy
k	shape parameter of Weibull distribution
c	scale parameter of Weibull distribution
P	wind turbine power
P_{aver}	average power output of the wind turbine
P_r	rated wind turbine power
x_1, x_2, x_3,	Cartesian coordinates
y_1, y_2, y_3	Cartesian coordinates

Greek symbols

α, β, γ	parameters
Γ	gamma function
Δt	time period
μ, ν	constants
σ	wind speed standard deviation
σ'	relative wind speed standard deviation
τ	capacity factor
τ_1, τ_2	terms of capacity factor
υ	wind speed
$\bar{\upsilon}$	mean wind speed
υ_{ci}	cut-in velocity
υ_r	rated velocity
υ_{co}	cut-out velocity
ϕ_1, ϕ_2, ϕ_3, ψ_1, ψ_2, ψ_3	functions

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